UNITARY SYMMETRY AND LEPTONIC DECAYS

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(Received 29 April 1963)

We present here an analysis of leptonic decays based on the unitary symmetry for strong interactions, in the version known as "eightfold way,"[1] and the $V-A$ theory for weak interactions.[2,3] Our basic assumptions on J_μ, the weak current of strong interacting particles, are as follows:

(1) J_μ transforms according to the eightfold representation of SU_3. This means that we neglect currents with $\Delta S=-\Delta Q$, or $\Delta I=3/2$, which should belong to other representations. This limits the scope of the analysis, and we are not able to treat the complex of K^0 leptonic decays, or $\Sigma^+ \rightarrow n+e^+ +\nu$ in which $\Delta S=-\Delta Q$ currents play a role. For the other processes we make the hypothesis that the main contributions come from that part of J_μ which is in the eightfold representation.

(2) The vector part of J_μ is in the same octet as the electromagnetic current. The vector contribution can then be deduced from the electromagnetic properties of strong interacting particles. For

$\Delta S = 0$, this assumption is equivalent to vector-current conservation.[2]

Together with the octet of vector currents, j_μ, we assume an octet of axial currents, g_μ. In each of these octets we have a current with $\Delta S = 0$, $\Delta Q = 1$ $j_\mu^{\{(0)\}}$ and $g_\mu^{\{(0)\}}$, and a current with $\Delta S = \Delta Q = 1$ $j_\mu^{\{(1)\}}$ and $g_\mu^{\{(1)\}}$. Their isospin selection rules are, respectively, $\Delta I = 1$ and $\Delta I = 1/2$.

From our first assumption we then get

$$J_\mu = a\left(j_\mu^{\{(0)\}} + g_\mu^{\{(0)\}}\right) + b\left(j_\mu^{\{(1)\}} + g_\mu^{\{(1)\}}\right).$$

A restriction $a = b = 1$ would <u>not</u> ensure universality in the usual sense (equal coupling for all currents), because if J_μ [as given in Eq. (1)] is coupled, we can build a current, $b\left(j_\mu^{\{(0)\}} + g_\mu^{\{(0)\}}\right) - a\left(j_\mu^{\{(1)\}} + g_\mu^{\{(1)\}}\right)$, which is not coupled. We want, however, to keep a weaker form of universality, by requiring the following:

(3) <u>J_μ has "unit length," i.e. ,</u> $a^2 + b^2 = 1$.

We then rewrite J_μ as[4]

$$J_\mu = \cos\theta\left(j_\mu^{\{(0)\}} + g_\mu^{\{(0)\}}\right) + \sin\theta\left(j_\mu^{\{(1)\}} + g_\mu^{\{(1)\}}\right),$$

where $\tan\theta = \mathrm{b} \,/\, \mathrm{a}$. Since J_μ, as well as the baryons and the pseudoscalar mesons, belongs to the octet representation of SU_3, we have relations (in which θ enters as a parameter) between processes with $\Delta S = 0$ and processes with $\Delta S = 1$.

To determine θ, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+\mu\nu)/\Gamma(\pi^+\mu\nu)$$

$$= \tan^2\theta M_K\left(1 - M_\mu^2/M_K^2\right)^2 / M_\pi\left(1 - M_\mu^2/M_\pi^2\right)^2.$$

From the experimental data, we then get[5, 6]

$$\theta = 0.257.$$

For an independent determination of θ, let us consider $K^+ \to \pi^0 + e^+ + \nu$. The matrix element for this process can be connected to that for $\pi^+ \to \pi^0 + e^+ + \nu$, known from the conserved vector-current hypothesis (2nd assumption). From the rate[6] for $K^+ \to \pi^0 + e^+ + \nu$, we get

$$\theta = 0.26.$$

The two determinations coincide within experimental errors; in the following we use $\theta = 0.26$.

We go now to the leptonic decays of the baryons, of the type $A \to B + e + \nu$. The matrix element of any member of an octet of currents among two baryon states (also members of octets) can be expressed in terms of two reduced matrix elements[7]

$$\langle A \mid j_\mu^{\{(i)\}} + g_\mu^{\{(i)\}} \mid B\rangle = \mathrm{i}f_{\mathrm{ABi}} O_\mu + d_{\mathrm{ABi}} E_\mu;$$

the f's and d's are coefficients defined in Gell-Mann's paper.[1, 7] It is sufficient to consider only allowed contributions and write

$$O_\mu, E_\mu = F^{\mathrm{O,\,E}}\gamma_\mu + H^{\mathrm{O,\,E}}\gamma_\mu\gamma_5.$$

From the connection with the electromagnetic current we get the vector coefficients: $F^O = 1$, $F^E = 0$; from neutron decay we get

$$H^O + H^E = 1.25.$$

We remain with one parameter which can be determined from the rate for $\Sigma^- \to \Lambda + e^- + \bar{\nu}$. The relevant matrix element for this is

$$\cos\theta\langle\Sigma^- \mid j_\mu^{\{(0)\}} + g_\mu^{\{(0)\}} \mid \Lambda\rangle$$

$$= \cos\theta\left(\frac{2}{3}\right)^{1/2} E_\mu = \left(\frac{2}{3}\right)^{1/2} \cos\theta H^E \gamma_\mu\gamma_5.$$

Taking the branching ratio for this mode to be 0.9×10^{-4},[8] we get

$$H^E = \pm 0.95.$$

The negative solution can be discarded because it produces a large branching ratio for $\Sigma^- \to n + e^- + \bar{\nu}$, of the order of 1%. The positive solution ($H^E = 0.95$, $H^O = 0.30$) is good, because it produces a cancellation of the axial contribution to this process. This explains the experimental result that this mode is more depressed than the $\Lambda \to p + e^- + \bar{\nu}$ in respect to the predictions of Feynman and Gell-Mann.[2]. In Table I, we give a summary of our predictions for the electron modes with $\Delta S = 1$. The branching ratios for $\Lambda \to p + e^- + \bar{\nu}$ and $\Sigma^- \to n + e^- + \bar{\nu}$ are in good agreement with experimental data.[9]

As a final remark, the vector-coupling constant for β decay is not $G\cos\theta$. This gives a correction of 6.6% to the ft value of Fermi transitions, in the right direction to eliminate the discrepancy between O^{14} and muon lifetimes.

Table 1: Predictions for the leptonic decays of hyperons

Decay	Branching ratio: From reference 2	Branching ratio: Present work	Type of interaction
$\Lambda \to p + e^- + \bar{\nu}$	1.4 %	0.75×10^{-3}	$V - 0.72A$
$\Sigma^- \to n + e^- + \bar{\nu}$	5.1 %	1.9×10^{-3}	$V + 0.65A$
$\Xi^- \to \Lambda + e^- + \bar{\nu}$	1.4 %	0.35×10^{-3}	$V + 0.02A$
$\Xi^- \to \Sigma^0 + e^- + \bar{\nu}$	1.14%	0.07×10^{-3}	$V - 1.25A$
$\Xi^0 \to \Sigma^+ + e^- + \bar{\nu}$	0.28%	0.26×10^{-3}	$V - 1.25A$

The correction is, however, too large, leaving about 2% to be explained.

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[1]M. Gell-Mann, California Institute of Technology Report CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958)

[2]R. E. Marshak and E. C. G. Sudarshan, Proceedings of the Padua-Venice Conference on Mesons and Recently Discovered Particles, September, 1957 (Società Italiana di Fisica, Padua-Venice, 1958); Phys. Rev. 109, 1860 (1958).

[4]Similar considerations are forwarded in M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1958).

[5]The lifetime from W. H. Barkas and A. H. Rosenfeld, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p.878. The branching ratio for $K^+ \to \mu^+ + \nu$ is taken as 57.4%. W. Becker, M. Goldberg, E. Hart, J. Leitner, and S. Lichtman (to be published).

[6]B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961). These authors give the branching ratio for $K^+ \to \mu^+ + \nu$ as 64%, from which $\theta = 0.269$. Also this value agrees with that from $K^+ \to \pi^0 + e^+ + \nu$ within experimental errors.

[7]N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1961). Our notation for the currents is different from the one used in this reference and by Gell-Mann; the connection is $j_\mu^0 = j_\mu^1 + ij_\mu^2$, $j_\mu^1 = j_\mu^4 + ij_\mu^5$.

[8]W. Willis et al. reported at the Washington meeting of the American Physical Society, 1963 [W. Willis et al. , Bull. Am. Phys. Soc. 8, 349 (1963] this branching ratio as $\left(0.9^{+0.5}_{-0.4}\right) \times 10^{-4}$. If it is allowed to vary between these limits, our predictions for the $\Sigma^- \to ne^-\overline{\nu}$ varies between 0.8×10^{-3} and 4×10^{-3}, and that for $\Lambda^0 \to pe^-\overline{\nu}$ between 1.05×10^{-3} and 0.56×10^{-3}. I am grateful to the members of this group for prepublication communication of their results.

[9]R. P. Ely, G. Gidal, L. Oswald, W. Singleton, W. M. Powell, F. W. Bullock, G. E. Kalmus, C. Henderson, and R. F. Stannard [Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 445] give the branching ratio for $\Lambda \to p + e^- + \overline{\nu}$ as $(0.85pm0.3) \times 10^{-3}$ while that for $\Sigma^- \to n + e^- + \overline{\nu}$ is given (see preceding reference) as $(1.9 \pm 0.9) \times 10^{-3}$.

[10]R. P. Feynman, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc. , New York, 1960), p. 501. Recent measurements of the muon lifetime have slightly increased the discrepancy. We think that more information will be needed to decide whether our 3rd assumption can be maintained.